CODING AND DECODING BY ITERATIVE FUNCTION SYSTEMS (IFS) WITH OSCILLATING COLLAGE FUNCTIONS AND A SPATIAL COLLAGE FUNCTION

The field of the invention is that of the encoding and especially the
5   compression of the volume of an image, with loss of information, for example for its
storage and/or its transmission.

The notion of image compression with loss is a concept well known in the
field of image processing. The main problem encountered is of course that of
obtaining an approximation of the original image that is as faithful as possible while
10   minimizing the volume of the approximation. To express the gain on a volume of
data, the general practice is to introduce the notion of the bit rate whose unit is one
bit per pixel (bpp). The bit rate therefore defines the ratio between the number of
pixels of the original image and the number of bits needed to reconstruct its
approximation.

15   The invention also relates to applications requiring image encoding at low bit
rates (for example at rates of below 0.3 bpp), for example applications on the
Internet. Of course, as shall be seen hereinafter, the invention can be applied to
many other technical fields.

Indeed, the invention provides a major improvement in fractal compression
20   techniques, also known as compression based on iterated function systems (IFS).
The principle of image compression by the IFS method relies on the expression of
the contents of the image by means of these contents themselves. It can therefore be
seen as a self-quantification of the image.

The formalizing of the IFS method has come especially through work by
25   Hutchinson, published in 1981 and by Bransley, Demko and others, researchers at
the Georgia Institute of Technology, between 1985 and 1988. The first automatic
algorithm applying these principles to image compression was proposed by Jacquin
in 1989.

The general principle of this algorithm relies on the partitioning (subdividing)
30   of the image I to be encoded into destination regions (also known as ranges). This
partitioning can be predefined or it can depend on the contents of the image. Then,
for each of the destination regions, the following operations are performed:

-    the choice (according to several known techniques) of another larger-
sized region of the image, not necessarily a partition element, known
35   as a source region (or domain);

- the determining of the spatial collage function, in a family of spatial collage functions fixed beforehand, that transforms the support of the source region so as to make it capable of being superimposed on the destination region to be considered.

5 If there is no corresponding function, there is a return to the previous step. Otherwise, the result of the application of the spatial collage function identified on the source region is called a decimated source region;

- determining the mass collage function, in the family of mass collage

10 functions fixed beforehand, which transforms the contents (for example the color and/or the gray level and/or at least one piece of photometric information) of the destination source region so as to make it as close as possible to that of the destination region considered.

15 If the proximity is not satisfactory, the first preceding step is resumed.

The set of spatial and mass collage functions thus chosen is called IFS. These spatial and mass collage functions are essentially refined functions. The term "collage function" is generally applied to the pair formed by the spatial collage function and the mass collage function applied to a source region.

20 Let I_R be an image with any contents, having the same support as the image I used to construct the IFS. The successive applications of IFS to I_R are used for the convergence (for example in about ten iterations) towards a "fixed point" that is an image I' close to I. This property is the basis of image-encoding by IFS. Indeed, it is enough to memorize the IFS in order to characterize I'.

25 However, while maintaining the general principle presented here above, much research has been done in order to improve the performance characteristics of the first IFS-based encoders.

In particular, it has been proposed to make adaptive partitions for the destination regions. Such partitions are used to pave the image with small regions on

30 textured zones that are difficult to approximate and to pave the image with large regions on the less dense zones which are easier to approximate.

Another advance is the hybridization of the IFS methods with other methods of image compression such as wavelets, vector quantification (VQ) or discrete cosine transform (DCT).

An IFS-wavelet hybridization consists of a search for similarity between the sub-bands of a breakdown into wavelets of the original image.

An IFS-VQ hybridization considers the IFS as a self-quantification of the image, and seeks to quantify the source regions of the IFS determined on the original image.

Finally, an IFS-DCT hybridization seeks for example to express the similarities of the coefficients of the DCT-transformed destination regions with source regions which too are transformed by DCT. An IFS-DCT hybridization of this kind may also consist of the application of an IFS computation to the residual image of a DCT reconstruction or again, conversely, the application of an approximation by DCT to the residual image of a reconstruction by IFS.

These different known techniques are relatively efficient in many situations. However, they have defects of analysis and synthesis, when they have to process zones with high frequencies and/or a textured content, especially when a very low bit rate is sought.

The goal of the invention especially is to overcome these drawbacks of the prior art.

More specifically, a goal of the invention is to provide an image-encoding method implementing IFS techniques that is more efficient, especially in terms of visual quality of approximation, and especially for the processing of high frequencies and textured zones while maintaining performance on the other zones.

Another goal of the invention is to provide an encoding method of this kind that is simple to implement with a reasonable quantity of computations. In particular, a goal of the invention is to provide an encoding method of this kind that can derive benefit from the advantages of the known techniques referred to here above in providing increased scalability.

It is also a goal of the invention to provide a corresponding decoding method which, as the case may be, can be parametrized.

These goals as well as others that shall appear hereinafter are achieved by means of an image-encoding method implementing iterated function systems (IFS), said method comprising the following steps:

- the partitioning an image I to be encoded into a set of image regions, known as destination regions, having an arbitrary shape (rectangular, triangular, or the like);

- the association, with each of said destination regions D, of a corresponding source region S and a collage function w such that w(S) is a good approximation of said destination region D,

said collage function being broken down into:

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- a spatial collage function w_S, acting on the position and/or the geometry of said source region S in order to create a decimated source region $\bar{S}$ (we have: $\bar{S} = w_S$ (S)); and

- a mass collage function w_M, acting on the contents (for example the color and/or the gray level and/or at least a photometrical information element) of said decimated source region $\bar{S}$.

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According to the invention, the mass collage function w_M is an oscillating function.

The invention therefore relies on a novel, inventive approach to collage. Indeed, whatever the technique used, it has always been considered that only the polynomial functions (which include especially the refined functions) could be used in this context.

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The use of oscillating functions is used to obtain efficient results especially for the processing of regions containing high frequencies. Thus, it is possible to faithfully reconstruct textures in avoiding the use of smoothing common to known IFS techniques.

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Advantageously, said mass collage function w_M is a harmonic function, and for example a cosine function.

In this case, a transformed source region S' = w(S) may advantageously be defined by:

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$$S'_i = w(S_i) = \sum_{l\in[0;N_c[} \sum_{k\in[0;N_c[} c_{kl} \times \bar{S}_i \times \cos(\theta_l i_x) \times \cos(\theta_k i_y) + b$$

where:

i is the index of the ith pixel of S', having co-ordinates (i_x, i_y);

$\bar{S}_i$ is the image of S_i with reference to w_S;

θ is a real vector of R^{N_c} such that $\theta_j = 2\pi/2^j$;

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c_{kl} and b are the coefficients characterizing collage function (the coefficients c_{kl} represent the amplitudes and b represents the shift).

Said coefficients c_{kl} and b may then be determined by searching for the coefficients minimizing an error between source and destination. This error E is written for example as follows:

$$E = \sum_{i \in [0; card(D)[} (S'_i - D_i)^2$$

with:

Card(D) being the number of pixels of D.

Various computation techniques may be used. In particular, it is possible to implement a matrix linear system whose approaches are determined by means of said methods belonging to the group comprising a:

- direct method;

- iterated method;

- gradient method.

According to a preferred embodiment of the invention, a direct Gauss pivot method or Cholesky pivot method is implemented.

Advantageously, said mass collage function w_M is written in the form of a combination of oscillating functions whose number and/or frequency and/or amplitude can be parametrized.

The invention also relates to an image encoding device implementing this method as well as the collage function itself, in which the mass collage function w_M is an oscillating function.

The invention also relates to the method of decoding images encoded by means of the encoding method described here above. According to this decoding method, said images are reconstructed by carrying out at least one iteration of said collage function applied to said corresponding source region S, said mass collage function w_M being an oscillating function.

According to a preferred embodiment of the invention, the mass collage function applied to said decimated source region during the decoding takes account of a number of oscillating functions smaller than or equal to the number taken into account during the encoding.

Thus, it is possible to carry out a progressive decoding and/or a scalability (ability to be sampled) during the decoding.

The invention also relates to data carriers containing images encoded according to the technique described here above (only said source regions S and said collage functions being stored on said data carrier).

The invention can be applied in many fields, especially in the fields belonging to the group comprising the following fields:

- compression of fixed images;

- compression (of images) in "intra" mode in a video encoder;
- compression of images or of a part of the images that are textured;
- magnification (zooming) of image zones;
- compression in spaces having a size greater than 2.

5   Other features and advantages of the invention shall appear more clearly from the following description of a preferred embodiment given by way of a simple illustrative and non-restrictive example, and the appended drawings which provide an illustration, in one example, of the principle of encoding by IFS:

- Figure 1 shows an original image to be compressed I;
10   - Figure 2 illustrates the construction of a partition on the original image of Figure 1;
- Figure 3 illustrates a search for a fixed region D of the region S and of the collage w that most closely approximates it;
- Figure 4 illustrates the determination of the collage function W of
15   Figure 3.

The invention therefore relates to an improvement of IFS-based techniques. It may be recalled that this technique, which is known per se, seeks to express the self-similarities of an image.

For example, figure 1 shows an original image that is to be compressed.

20   In the first processing step, a partition of the original image is made, as shown in figure 2, in the form of square-shaped regions or blocks. The partitioning can also be done from another basic pattern, especially triangles. It can also be adaptive, i.e. it can take account of the contents of the image, and especially of the complexity of the different parts of this image.

25   Then, for each region D31 of the partition, a search is made for the region S (32, 33 or 34) and the collage w that most closely approximates it, as is shown in figure 3.

In the example shown, it is the region S_2 33 that meets this criterion.

Figure 4 illustrates the principle of the collage function w. This collage
30   comprises first of all a spatial collage w_s, that shifts, decimates and orients the region

33 so that it geometrically approaches the region 31. Then, the mass collage function w_M is used to obtain the region 41 S'_2, approximating the region 31.

More specifically, when we consider the destination region D 31, the resolution consists in characterizing a source region 33 as well as a collage function w, such that w(S) is a close approximation of S. In a particular case, if S'=(S) and if the error measurement considered is the distance L_2, then the error to be minimized is defined by:

$$E = \sum_{i \in [0;card(D)[} (S'_i - D_i)^2$$

where S'_i (and D_i respectively) is the intensity of the ith pixel of S' (and D respectively), and where card(D) is the number of pixels of D.

The collage function can be subdivided into two sub-functions. The first is the spatial collage w_s which acts on the position and geometry of the regions. It transforms the source region into a decimated region $\overline{S}$. The second is the mass collage w_M which acts on the contents of the regions. It transforms the decimated source region $\overline{S}$ into a region S' most closely approximating the region D considered in the sense of the error defined here above.

The invention relates to the nature of the mass collages w_M.

According to the known techniques, these operations of mass collage w_M are polynomial functions (which include especially the refined functions). More specifically, they bring into play two variables α and β called a scale factor and an offset factor.

It has also been envisaged to use polynomial mass collages or collages defined by function trees. However, these techniques have proved to be inefficient or highly complex, and, therefore, especially, unsuited to image compression in grey levels.

According to the invention, oscillating mass collage functions are used. In the embodiment described here below, by way of an example, cosine collage functions are used.

This approach introduces high frequencies into the IFS collages, when the contents of the source regions is not sufficiently dense. The collage of the invention optimizes the characterization of the source regions in conjunction with the coefficients associated with the cosine functions. Thus, the source regions provide the low frequency information as well as the breaks in slope such as the contours, that are enriched by the cosine functions, if the destination region considered comprises high frequency information.

If we consider a destination region D and a source region S, it is possible to conventionally determine a spatial collage w_S. The image $\bar{S}$ is called the image of S by w_s.

The contents of the transformed source region S' approximating D can be expressed, according to the approach of the invention, as follows:

$$S'_i = w(S_i)$$
$$S'_i = w_M(w_S(S_i))$$
$$S'_i = w_M(\bar{S}_i)$$
$$S'_i = \sum_{l \in [0;N_c[} \sum_{k \in [0;N_c[} c_{kl} \times \bar{S}_i \times \cos(\theta_l i_x) \times \cos(\theta_k i_y) + b$$

where: i is the index of the ith pixel of S', having coordinates (i_x, i_y) ;

$\bar{S}_i$ is the image of S_i by w_s ;

θ_j is a real vector of R^{Nc} such that $\theta_j = 2\pi/2^j$;

c_{kl} and b are coefficients characterizing the collage function.

The characterization of w_M is obtained by the determining of the values of amplitude of the cosines represented by a matrix, and of the offset b, represented by a scalar value.

These values must minimize the collage error E between S' and D.

The expression to be minimized is therefore the following:

$$\min E \;=\; \min \sum_{i\in[0;card(D)[} (S'_i - D_i)^2$$

The minimum of this error E, which is strictly positive, may be obtained for the coefficients c_{kl} and b which cancel the derivative of E. The problem is therefore expressed by the resolution of the following system of $(N_c^2 + 1)$ rows and $(N_c^2 + 1)$ unknown quantities:

$$\left\{ \begin{array}{l} \vdots \\[4pt] \dfrac{\partial E}{\partial c_{00}} = 0 \\[8pt] \ldots \\[4pt] \dfrac{\partial E}{\partial c_{ij}} = 0 \\[8pt] \ldots \\[4pt] \dfrac{\partial E}{\partial c_{N_c-1;N_c-1}} = 0 \\[8pt] \dfrac{\partial E}{\partial b} = 0 \end{array} \right.$$

After the development of the expressions, this system can be rewritten in the form of a matrix linear system.

$$A\begin{pmatrix} A_{00}^{00} \cdots A_{0N_c-1}^{00} \; A_{10}^{00} \cdots\cdots A_{N_c 0}^{00} \\ \vdots \\ A_{00}^{0N_c} \\ A_{00}^{1N_c} \\ \vdots \\ A_{00}^{N_c N_c} \cdots A_{0N_c-1}^{N_c N_c} \; A_{10}^{N_c N_c} \cdots A_{N_c 0}^{N_c N_c} \end{pmatrix} * X\begin{pmatrix} c_{00} \\ \vdots \\ \vdots \\ \vdots \\ c_{N_c-1 N_c-1} \\ b \end{pmatrix} = B\begin{pmatrix} B_{00} \\ \vdots \\ \vdots \\ B_{N_c-1 N_c-1} \\ B_{N_c-1 N_c} \end{pmatrix}$$

Where $A \in \mathcal{M}^{N_c^2+1}(\mathfrak{R})$ is a known expression defined as follows :

$$\left\{ \begin{array}{l} A_{ij}^{kl} = \displaystyle\sum_{m\in[0..card(D)[} \overline{S}_m^2 * \Psi_m^{ij} * \Psi_m^{kl} \;,\quad \forall\, i\in[0..N_c[\,,\; j\in[0..N_c[\,,\; k\in[0..N_c[\,,\; l\in[0..N_c[\\[14pt] A_{ij}^{N_c N_c} = \displaystyle\sum_{m\in[0..card(D)[} \overline{S}_m * \Psi_m^{ij} \;,\quad \forall\, i\in[0..N_c[\,,\; j\in[0..N_c[\\[14pt] A_{N_c 0}^{kl} = \displaystyle\sum_{m\in[0..card(D)[} \overline{S}_m * \Psi_m^{kl} \;,\quad \forall\, k\in[0..N_c[\,,\; l\in[0..N_c[\\[14pt] A_{N_c 0}^{N_c N_c} = card(D) \end{array} \right.$$

Taking $\Psi_m^{ij} = \cos(\theta_i * m_x) * \cos(\theta_j * m_y)$

with $m_x;m_y$ being the local co-ordinates of the mth pixel of D

5   Where $B \in \mathfrak{R}^{N_c^2+1}$ is a known vector, defined as follows :

$$\begin{cases} B_{ij} = \sum_{m \in [0..card(D)[} \overline{S}_m * D_m * \Psi_m^{ij} & pour \; i \in [0..N_c[\quad j \in [0..N_c[\\ B_{N_c-1N_c} = \sum_{m \in [0..card(D)[} D_m \end{cases}$$

This system can be resolved by different methods (direct method, iterated

10   method, gradient method etc). In particular, it is possible to use the GAUSS pivot

method.

Efficient encoding results are obtained with a very small number of

coefficients. Two or three basic oscillating functions already give good results.

Naturally, the greater this number, the higher is the quality.

15   During the decoding, it is possible to take account of a smaller number of

oscillating functions. With this, it is possible to take account of the processing

capacity of the receiver or of the user's needs, but also to carry out a gradual

decoding (the image being first of all reconstructed with average quality and then

being gradually refined). This also gives scalability during the decoding.

20   The technique of the invention can be used in very many fields, including that

of image compression, especially when it has some textured parts. The invention also

makes it possible to obtain zoom operations, by modification if the partition at

decoding.

The invention can be applied also to image processing in spaces with a

25   dimension of over 2 (for example in video (2D+t) or in virtual images (3D)). It can

also be implemented for one-dimensional images